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Supermultiplets of Elementary Particles*

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The notion of supermultiplets first developed by Wigner for the theory of nuclear structure is applied to the structure of elementary particles. The group structure is assumed to be SU_6. The quark model is assumed for the entire discussion, although some of these results can be obtained from other models. It is found that the octet of pseudoscalar mesons along with the octet and the singlet of vector mesons form a supermultiplet. Okubo's speculated mass form for the vector mesons is derived. It is also found that the octet of baryons along with a singlet particle of spin $\frac{3}{2}$ form a supermultiplet. The type of baryonic coupling for the electromagnetic and weak current is derived.

THE supermultiplet theory of the nucleus has been proposed by Wigner.[1] If one neglects the Coulomb interaction, nuclear forces are invariant under rotations in isospin space. If one also neglects the tensor force and the spin-orbit force, the spin variables and space variables are decoupled in the interactions so that nuclear forces are invariant under a group SU_2 which transforms the nucleon-spin wave functions only among themselves. Thus, the nuclear interactions are invariant under a group $SU_2 \otimes SU_2$ which is a direct product of two SU_2 groups: the ordinary spin group and the isospin group. If the main part of the nuclear forces contributing to nuclear binding does not depend on spin and isospin variables at all, it is invariant under a larger group of transformations, SU_4, which transforms four fundamental constituents among themselves: $p_\uparrow$, $p_\downarrow$, $n_\uparrow$, and $n_\downarrow$. A basis of an irreducible representation of the group SU_4 is said to characterize a supermultiplet of nuclear levels. In general, it can be reduced to bases of irreducible representations of a subgroup of the SU_4, the group $SU_2 \otimes SU_2$. By this reduction it can be seen what spin and charge multiplets of nuclear levels belong to a supermultiplet. One of the characteristics of the supermultiplet theory is that different spin multiplets as well as charge multiplets can be in a same supermultiplet, as the n-p systems in ${}^3S_1$ and ${}^1S_0$ state are.

The existence of an analogy between the elementary particle structure and the nuclear structure has been stressed by Sakata,[2] and he proposed a model in which every elementary particle is a bound state of members of the fundamental triplet p, n, and Λ; this is known as the Sakata model.

A group of unitary transformations which transform components of a fundamental triplet among themselves is known as a SU_3 group.[3] The octet model (eightfold way) based on the group SU_3 has been proposed by Gell-Mann and Ne'eman[4] and has considerable support from experiments, although at first sight it seems not to have an analogy between the elementary-particle structure and the nuclear structure. As pointed out by Gell-Mann and Zweig,[5] however, this analogy may be retained simply by introducing a fundamental triplet whose members, called 'quarks,' have fractional electric and baryonic charges.[6]

In what follows we assume a fundamental triplet of "quarks" of spin $\frac{1}{2}$ as the constituents of elementary particles just as the proton and the neutron constitute nuclei. Further, we assume that the interaction binding the fundamental triplets to form an elementary particle are predominantly spin-independent. Then we may proceed with the discussion of the elementary-particle structure in a way parallel to the nuclear structure mentioned before. The group we are going to work with is the group SU_6 which transforms the six components of a fundamental triplet (including its spin components) among themselves. We then obtain a supermultiplet of elementary particles as a basis of an irreducible representation of the group SU_6, which can be reduced into bases of irreducible representations of a subgroup of the SU_6, the group $SU_2 \otimes SU_3$, where the SU_2 refers to the ordinary spin group and the SU_3 refers to the familiar SU_3 group. By this reduction we may exhibit the members of a supermultiplet in terms of the familiar SU_3 multiplets. The larger group SU_6 mixes the spin and SU_3 spin coordinates so that particles with different spin as well as with different isospin and strangeness can be in a same supermultiplet. As will be seen, the pseudoscalar mesons and the vector mesons are in a same supermultiplet in this theory.

One of the most remarkable successes of the SU_3 symmetry theory is the existence of the Gell-Mann—Okubo (GMO) mass formula[7] which may be obtained

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[1] E. Wigner, Phys. Rev. **51**, 106 (1937).
[2] S. Sakata, Progr. Theoret. Phys. (Kyoto) **16**, 686 (1956).
[3] M. Ikeda, S. Ogawa, and Y. Ohnuki, Progr. Theoret. Phys. (Kyoto) **22**, 715 (1959); **23**, 1073 (1960).
[4] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).
[5] M. Gell-Mann, Phys. Letters, **8**, 214 (1964); G. Zweig (unpublished).
[6] Another possible model which retains the analogy of the nuclear structure is a broken SU_4 model; Y. Hara, Phys. Rev. **134**, B701 (1964); Z. Maki, Progr. Theoret. Phys. (Kyoto) **31**, 331 (1964).
[7] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).

from a symmetry-breaking Hamiltonian transforming as a component $T_3{}^3$ of the regular representation of the SU_3 group. Here we shall assume two kinds of symmetry-breaking interactions: the one is SU_6-noninvariant but $SU_2 \otimes SU_3$-invariant and the other is SU_3-noninvariant. We shall also assume that the latter interaction transforms as a component of the regular representation of SU_6 as well as SU_3; this is a simple generalization of the Okubo's symmetry-breaking interaction to our theory.

A basis of an irreducible representation of the SU_6 group can be obtained by considering an irreducible tensorial set in a six-dimensional complex-vector space. In general, the irreducible tensor can be designated by a signature of the Young symmetrization operation.[8] As far as the irreducible representations of lower dimensions are concerned, however, it is simpler, more physical, and more transparent to use for the basis of the representation an elementary tensor, which can be interpreted as a wave function of a system. We shall write these tensors by using capital letters for their suffixes running from 1 to 6. Since we will decompose the irreducible representation of the SU_6 into the irreducible representations of the $SU_2 \otimes SU_3$ sooner or later, to see the members of a supermultiplet, it is convenient to assign a set of suffixes $i\alpha$ for each A, where a Latin letter (i) runs 1 and 2 to form a tensor representation of the group SU_2 while a Greek letter (α) runs from 1 to 3 for the group SU_3.

To denote an irreducible representation of the SU_6 group, we shall also use the dimension of the representation as is the prevailing custom in SU_3 theory. For example, "6" denotes a fundamental representation, "6*" denotes its conjugate representation and "35" denotes a regular representation, etc. To denote an irreducible representation of the group $SU_2 \otimes SU_3$, we use a pair of numbers (a,α), where a and α denote the dimension of the representation of the SU_2 and the SU_3 groups, respectively; thus the spin of a particle belonging to this representation is given by $\frac{1}{2}(a-1)$. By the help of this notation, the reduction of an irreducible representation of SU_6, "A," into irreducible representations of $SU_2 \otimes SU_3$, (a,α), (b,β), $\cdots$, is written as following:

$$\text{“}A\text{”} = (a,\alpha)+(b,\beta)+\cdots$$

in which an arithmetic relation $A = a\alpha + b\beta + \cdots$ must be satisfied.

After these preparations we shall first consider the pseudoscalar and vector mesons, which we assume are bound states of a fundamental particle and an antiparticle in the ${}^1S_0$ and ${}^3S_1$ states, respectively. Since the fundamental particle belongs to a fundamental representation of the SU_6, "6," and its antiparticle belongs to its conjugate representation, "6*," a bound state of these particles must belong to the product representation of the SU_6, "6"$\times$"6*", which can be reduced to "1" and "35": "6"$\times$"6*"="35"+"1". It is not difficult to see that the representation "35" is reduced into (1,8), (3,8) and (3,1) representations of the group $SU_2 \otimes SU_3$, i.e., "35"=(1,8)+(3,1)+(3,8), with which we may identify the octet of pseudoscalar mesons, the singlet and the octet of vector mesons, respectively. In this theory, therefore, the octet of pseudoscalar mesons $(K^+K^0\pi^+\pi^-\pi^0K^-\bar{K}^0\eta)$ and the singlet and the octet of vector mesons $(\phi\omega K^{+*}K^{0*}\rho^+\rho^0\rho^-K^{-*}\bar{K}^{0*})$ belong to the same supermultiplet. By using the tensor notation, the basis of the "35" representation of the group SU_6 can be specified by a traceless second-rank mixed tensor $\Phi^A[\Phi_A{}^A=0]$. The reduction stated above can be demonstrated explicitly for the meson wave function using elementary tensor calculus:

$$\Phi_B{}^A \equiv \Phi_{j\beta}{}^{i\alpha} = (1/\sqrt{2})[\delta_j{}^i f_\beta{}^\alpha + (\boldsymbol{\sigma})_{ij} \times\{\mathbf{V}_\beta{}^\alpha + (1/\sqrt{3})\delta_\beta{}^\alpha \mathbf{V}_0\}], \quad (1)$$

where $f_\beta{}^\alpha$, $\mathbf{V}_\beta{}^\alpha$, and $\mathbf{V}_0$ are an octet pseudoscalar-meson, an octet vector-meson, and a singlet vector-meson wave function, respectively, and $\boldsymbol{\sigma}$ is the usual Pauli spin matrix.

With the singlet representation "1" we may identify the recently discovered X^0 meson[9]; it belongs to a different supermultiplet here, in contrast to other models.[10]

As mentioned before, we shall assume that the total Hamiltonian of a system has the following three parts:

(i) The SU_6 invariant part H_0.

(ii) The SU_6 noninvariant but $SU_2 \otimes SU_3$ invariant part H'. This interaction H' is spin-dependent and/or SU_3 spin-dependent, and in general it gives mass splitting between particles of different ordinary spin and/or SU_3 spin.

(iii) The SU_3 noninvariant part H''. We assume that H'' transforms as a regular representation of the SU_6 as well as the SU_3 group, and the transformation property of H'' for the SU_3 group is $T_3{}^3$ in order to retain the GMO relation. Using the tensor notation, H'' can be written as

$$H'' = T_{i3}{}^{i3}, \quad (2)$$

where $T_B{}^A$ is a component of a representation "35". This is the simplest generalization of the GMO symmetry-breaking Hamiltonian.

For this Hamiltonian we may obtain the form of the mass term in an effective Lagrangian for the pseudoscalar mesons and the vector mesons; it has three parts corresponding to the three parts of the Hamiltonian. The contribution of H' to an effective-mass

[8] For example, H. Weyl, *The Classical Group* (Princeton University Press, Princeton, New Jersey, 1946).

[9] G. R. Kalbfleisch, L. W. Alvarez, A. Barbaro-Galtieri, O. I. Dahl *et al.*, Phys. Rev. Letters **12**, 527 (1964); M. Goldberg, M. Gundzik, S. Lichtman, J. Leitner *et al.*, *ibid.* **12**, 546 (1964).

[10] J. Schwinger, Phys. Rev. **135**, B816 (1964); F. Gursey, T. D. Lee, and M. Nauenberg, *ibid.* **135**, B467 (1964).

term must have the following form:

$$m_1^2\Phi_{i\beta}{}^{i\alpha}\Phi_{j\alpha}{}^{j\beta}+m_2^2\Phi_{i\alpha}{}^{i\alpha}\Phi_{j\beta}{}^{j\beta}. \tag{3}$$

The first term of (3) gives a spin-dependent mass splitting while the second term is SU_3 spin-dependent. It is more transparent if we use the explicit formula for Φ, Eq. (1), in (3), giving

$$2m_1^2 f_\beta{}^\alpha f_\alpha{}^\beta+3m_2^2\mathbf{V}_0\cdot\mathbf{V}_0. \tag{3$'$}$$

To see the mass splitting due to H'' we shall construct the effective-mass term which transforms as Eq. (2):

$$m_3^2\{\Phi_A{}^{i3}\Phi_{i3}{}^A-\tfrac{1}{3}\Phi_B{}^A\Phi_A{}^B\}, \tag{4}$$

which can be written as

$$m_3^2(f_\alpha{}^3 f_3{}^\alpha+\mathbf{G}_\alpha{}^3\cdot\mathbf{G}_3{}^\alpha), \tag{4$'$}$$

where $\mathbf{G}_\alpha{}^\beta=\mathbf{V}_\alpha{}^\beta+(1/\sqrt{3})\delta_\alpha{}^\beta\mathbf{V}_0$, and we omitted the contribution of the second term in (4) because it does not affect the mass differences. The first term in (4$'$), $f_\alpha{}^3 f_3{}^\alpha$, gives the GMO mass relation for the pseudoscalar mesons while the second term, $G_\alpha{}^3G_3{}^\alpha$, gives a $\varphi-\omega$ mixing as well as a mass splitting among the vector mesons. The form of the second term is exactly the same as the Okubo's speculated expression for the vector mesons.[11] Therefore, if we neglect the SU_3 spin-dependent term [second term in (3$'$)], we obtain the equal-spacing result for the vector mesons:

$$m_\phi^2-m_{K^*}^2=m_{K^*}^2-m_\rho^2 \quad\text{and}\quad m_\rho^2=m_\omega^2.$$

Since the pseudoscalar and vector mesons belong to the same supermultiplet, the mass splittings among the octet of pseudoscalar mesons and among the octet of vector mesons are related, as seen explicitly in (4$'$), from which we obtain for example

$$m_{K^*}^2-m_\rho^2=m_K^2-m_\pi^2,$$

which is known to be satisfied.[12]

We shall assume that the baryons are composed of three fundamental particles in the S state. Because of the Pauli principle, the three-particle wave function must be antisymmetric for the interchange of any pair of the particles so that it is a basis of the irreducible representation "20" of the SU_6. The representation "20" reduces to the (2,8) and the (4,1) representation of $SU_2\otimes SU_3$. With the representation (2,8) we may identify the octet of baryons $(p,n,\Sigma^+,\Sigma^0,\Sigma^-,\Xi^-,\Xi^0,\Lambda^0)$, while with the (4,1) we cannot identify any known particle. The particle, tentatively denoted by Z^0, belonging to the representation $(4\cdot1)$ has spin and parity $\frac{3}{2}^+$, isospin 0, and strangeness -1, so that it should be observed as a $\Sigma\pi$ resonance in the $P_{3/2}$ state.

The wave function of the system can be written as a totally antisymmetric third-rank tensor in a six-dimensional complex-vector space: Ψ_{ABC}, where $\Psi_{ABC}=-\Psi_{BAC}=-\Psi_{ACB}=-\Psi_{CBA}$. The decomposition of the Ψ into the $(2\cdot8)$, $N_{\alpha i}{}^\beta$, and the $(4\cdot1)$, $\psi_{(ijk)}$, can be exhibited as follows:

$$\begin{aligned}\Psi_{ABC}\equiv\Psi_{i\alpha,j\beta,k\gamma}=&(1/\sqrt{6})\psi_{(ijk)}\epsilon_{\alpha\beta\gamma}\\&+(1/3\sqrt{6})[\epsilon_{jk}\{\epsilon_{\alpha\beta\delta}N_{\gamma i}{}^\delta+\epsilon_{\alpha\gamma\delta}N_{\beta i}{}^\delta\}\\&+\epsilon_{ki}\{\epsilon_{\beta\gamma\delta}N_{\alpha j}{}^\delta+\epsilon_{\beta\alpha\gamma}N_{\gamma j}{}^\delta\}\\&+\epsilon_{ij}\{\epsilon_{\gamma\alpha\delta}N_{\beta k}{}^\delta+\epsilon_{\gamma\beta\delta}N_{\alpha k}{}^\delta\}],\end{aligned} \tag{5}$$

where $\psi_{(ijk)}$ is completely symmetric with respect to the interchange of the suffixes so that it is a spin wave function of a spin-$\frac{3}{2}$ particle, and ϵ_{ij} and $\epsilon_{\alpha\beta\gamma}$ are the totally antisymmetric tensors in the two- and three-dimensional spaces, respectively.

A mass formula for the baryons can be obtained in the same way as the mesons. However, it is obvious that no more than the GMO mass formula is obtained for the baryons. Unfortunately, the mass of Z^0 may not be predicted because it depends on the spin-dependent mass splitting which is unknown *a priori*.

In order to see the transformation property of the electromagnetic interaction, we write down the minimal electromagnetic interaction of the fundamental particle in the static limit as the following:

$$\begin{aligned}e[\psi^{*i1}\psi_{1i}-\tfrac{1}{3}\psi^{*i\alpha}\psi_{i\alpha}]\phi&\\+(-ie/2m)(\psi^{*i1}\overleftrightarrow{\nabla}\psi_{i1}-\tfrac{1}{3}\psi^{*i\alpha}\overleftrightarrow{\nabla}\psi_{i\alpha})&\\+(e/2m)(\psi^{*1}\boldsymbol{\sigma}\psi_1-\tfrac{1}{3}\psi^{*\alpha}\boldsymbol{\sigma}\psi_\alpha)\times\nabla\cdot\mathbf{A},&\end{aligned} \tag{6}$$

where $\psi_{i\alpha}$ is the wave function of the fundamental particle, and ϕ and $\mathbf{A}$ are the electromagnetic scalar and vector potentials, respectively. The first two terms in (6) comprise the electric interaction, which transforms as a member of the "35" representation under the SU_6 group and the (1,8) representation under the $SU_2\otimes SU_3$ group. The third term is the magnetic interaction which transforms as a member of "35" under SU_6 and (3,8) under $SU_2\otimes SU_3$. We shall construct the effective electromagnetic current of baryons in a bilinear form of Ψ in such a way to retain these transformation properties. Since "20"$\times$"20*" contains only one "35" ("20"$\times$"20*"="1"+"35"+"175"+"189"), we expect only one type of coupling for the magnetic current as well as for the electric current, while in the ordinary SU_3 symmetry theory there are, in general, two types of coupling for the magnetic current known as F type and D type.[13]

The form of the tensor of the "35" representation constructed as a product of the "20" baryon and the "20*" baryon* wave function is given by

$$J_B{}^A=\Psi^{*ACD}\Psi_{BCD}-\tfrac{1}{6}\delta_B{}^A\langle\Psi^*\Psi\rangle, \tag{7}$$

where $\langle\Psi^*\Psi\rangle$ is an abbreviation for the trace, i.e., $\langle\Psi^*\Psi\rangle=\Psi^{*ABC}\Psi_{ABC}$. Inserting the expression for the Ψ, Eq. (5), into Eq. (7) to see the type of the coupling of the octet baryonic current, we obtain the following

[11] S. Okubo, Phys. Letters, **5**, 165 (1963).
[12] S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964).
[13] S. Coleman and S. L. Glashow, Phys. Rev. Letters **6**, 423 (1961).

expression for the $J_B{}^A$ after a lengthy but straightforward calculation:

$$\begin{aligned}J_B{}^A\equiv J_{j\beta}{}^{i\alpha}=&\tfrac{1}{6}\delta_\beta{}^\alpha(\boldsymbol{\sigma})_{ij}\cdot\psi^{*(klm)}(\boldsymbol{\sigma})_{kn}\psi_{(nlm)}\\&+\tfrac{1}{3}[\psi^{*(ikl)}\epsilon_{lj}N_{\beta k}{}^\alpha+\bar{N}_\beta{}^{\alpha k}\epsilon^{li}\psi_{(jkl)}]\\&+\tfrac{1}{6}[\delta_j{}^i(\bar{N}N_F)_\beta{}^\alpha-(\boldsymbol{\sigma})_{ij}\{(\bar{N}\boldsymbol{\sigma}N_D)_\beta{}^\alpha\\&+\tfrac{2}{3}\delta_\beta{}^\alpha\langle\bar{N}\boldsymbol{\sigma}N\rangle],\end{aligned}\qquad(7')$$

where

$$\begin{aligned}(\bar{N}N_F)_\beta{}^\alpha&=\bar{N}_\gamma{}^\alpha N_\beta{}^\gamma-\bar{N}_\beta{}^\gamma N_\gamma{}^\alpha,\\(\bar{N}N_D)_\beta{}^\alpha&=\bar{N}_\gamma{}^\alpha N_\beta{}^\gamma+\bar{N}_\beta{}^\gamma N_\gamma{}^\alpha-\tfrac{2}{3}\delta_\beta{}^\alpha\langle\bar{N}N\rangle,\\\langle\bar{N}N\rangle&=\bar{N}_\beta{}^\alpha N_\alpha{}^\beta.\end{aligned}$$

The last line in Eq. (7′) is the octet baryonic current and the terms in this line correspond to the (1,8), the (3,8), and the (3,1) representation of the $SU_2\otimes SU_3$ group, respectively. From this expression we may see that we have F-type coupling for the (1,8) current while we have D-type coupling for the (3,8) current.

Recalling that the electric and the magnetic interactions transform as the (1,8) and the (3,8) representation, respectively, we conclude from the previous considerations that the electric interaction of the octet of baryons is of F-type coupling, as we would expect from charge conservation, while the magnetic interaction is of D type.

We may readily extend above observations to the weak baryonic current in the leptonic decays of the baryons. We shall assume that the main part of the weak current is a bilinear form of the fundamental particle fields. Then the weak currents belong also to the "35" representation of the SU_6 group in the static limit. Considering the spin dependence of the currents, the Fermi current belongs to the (1,8) representation while the Gamor-Teller current belongs to the (3,8) representation of the $SU_2\otimes SU_3$ group. By using the previous observations, therefore, we conclude that the Fermi current must be of the F type and Gamow-Teller current must be of the D type. In the V-A theory of the weak interaction, the V current gives the Fermi current while the A current gives the Gamow-Teller current. Therefore, we may see that the conclusion we derived is consistent with present experiments by comparing with Cabibbo's work[14] in which he showed that the V current was of F type and the A current was almost pure D type.

If we assume that the conservation of parity holds for strong interactions, a nonderivative type of static meson baryon interaction is not allowed. But once we introduce a derivative into the interactions it can no longer be invariant under the group SU_6. Therefore the static meson-baryon interaction is a symmetry-violating interaction of the SU_6, although it is still invariant under the group SU_3.

Let us first consider the interaction of mesons with the fundamental particles in a static limit which is given by

$$g_0\psi^{*\alpha}\boldsymbol{\sigma}\psi_\beta\cdot\nabla f_\beta{}^\alpha.$$

In this interaction the $\psi^{*\alpha}\sigma\psi_\beta$ part transforms as the "35" representation of the SU_6 and the (3,8) representation of the $SU_2\otimes SU_3$. We may obtain an effective meson-baryon interaction from this interaction Hamiltonian by taking the expectation value of it between two baryonic states. If we assume that the meson field is not affected in taking an expectation value and is treated as an external field as in the case of the electromagnetic field, we see that the meson field interacts with baryons in the same way as the electromagnetic field does. Therefore, we obtain the D-type coupling for the octet meson baryon interactions which seems to be consistent with the analysis of the $P_{3/2}$ meson-baryon resonances.[15]

We may make an analogous discussion for the octet vector-meson–baryon interactions and find that F-type coupling holds for them.

We assumed the quark model of elementary particles and applied the idea of the supermultiplet of the nucleus to it without any justification. The results obtained seem to be interesting and give some support to the idea of a supermultiplet of elementary particles. Although the quark model is assumed here, the idea of supermultiplet may still be useful for other models such as the broken SU_4 model in which the same supermultiplet of mesons must be obtained.

Since we started from the quark model, we should not have R invariance at the beginning. However, if we look at the effective interactions of the baryons and the mesons, we see the existence of the R invariance in it if we appropriately define the R transformation for the octet pseudoscalar and vector mesons (RC of the mesons$=1$). This suggests, in general, that even if the original model does not indicate the R invariance, an approximate stronger symmetry (in this case the group SU_6) is possible to impose the R invariance as a result in the same approximation.

Our discussion was in a nonrelativistic framework. One would find some difficulty in extending this theory to satisfy the relativistic dynamics. As we mentioned before, even in the nuclear supermultiplet theory it is necessary to separate the spin group from the ordinary space-rotation group, which was accomplished by neglecting the tensor and the spin-orbit interaction in a first approximation. In a relativistic wave equation, the spin indices are so tightly related to the coordinates (as we may see in the Dirac equation) that it is impossible to decouple the spin variables from the coordinates in the free Hamiltonian. Therefore, in order to proceed with the same arguments as given here in the case of relativistic dynamics, we might have to separate a part of the free Hamiltonian from the rest in a first approximation and treat it as a perturbation,

[14] N. Cabibbo, Phys. Rev. Letters, **10**, 531 (1963).

[15] A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963); R. Cutkosky, Ann. Phys. (N. Y.) **23**, 415 (1963).

which resembles a treatment in the old strong-coupling theory.

Another possibility of extension is to search for a larger group which includes the SU_6 group and the Lorentz group as subgroups. One possible group of this kind is the SL_6 group (unimodula linear-transformation group in a six-dimensional complex-vector space), which contains $SL_2 \otimes SL_3$ as a subgroup. However, it seems to be impossible to have a wave equation compatible with this group without extending the four-dimensional Minkowsky space to a higher dimensional space (36-dimensional space!).[16]

[16] After this paper was written, the author became aware of a paper by F. Gürsey and L. A. Radicati [Phys. Rev. Letters **13**, 173 (1964)], in which they claim that the relativistic extension of the theory is possible.

PHYSICAL REVIEW VOLUME 136, NUMBER 6B 21 DECEMBER 1964

Self-Consistent Calculation for the Nucleon Mass, π-N Coupling Constant, and the Low-Energy $P_{1/2}$, $T=\frac{1}{2}$, π-N Scattering Phase Shifts

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A self-consistent calculation is performed for the low-energy $P_{1/2}$, $T=\frac{1}{2}$, pion-nucleon scattering amplitude, using the N/D method. The distant part of the left-hand cut for the partial-wave amplitude is dealt with in the manner of Balázs. The mass of the nucleon bound state is found to be ≈ 880 MeV, the coupling constant ≈ 12. The low-energy phase shifts and the scattering length are also in good qualitative agreement with the observed ones.

I. INTRODUCTION

DURING the past few years, the low-energy pion-nucleon system has been subjected to extensive investigations within the framework of dispersion theory. The main feature of such investigations has been to calculate the pion-nucleon scattering amplitude (to be precise, the various low angular-momentum partial-wave amplitudes) given by forces that arise from the low-mass intermediate states in the crossed $\pi - N$ and $\pi\pi \to N\bar{N}$ channels.[1,2] These low-mass states are the familiar nucleon and the $P_{3/2}$, $T=\frac{3}{2}$, $\pi - N$ resonance, N^* for the crossed $\pi - N$ channel and the $T=1$, $J=1$, $\pi\pi$ resonance, i.e., the ρ meson, for the channel $\pi\pi \to N\bar{N}$. For a more accurate determination of the $\pi - N$ scattering amplitude, one must of course incorporate in the problem the forces that arise from the higher mass intermediate states in the crossed channels. In the language of the N/D method, that provides the appropriate technique for such calculations, this amounts to taking into account the contributions that arise from the distant part of the left-hand cut of the various partial-wave amplitudes in the s plane. This, however, is a difficult problem. A method to tackle it has been suggested by Balázs,[3] wherein these far left-hand cut contributions can be approximated by a set of pole terms; the locations of these poles are known but the residues are, as such, unknown constants. However, if one knows the amplitude correctly at any point in the low-energy region, these residues can be determined by matching, at this point, the expression for the amplitude involving these residues with the known amplitude.[4] The latter quantity can be obtained from the absorptive parts of the crossed channels through a fixed-energy dispersion relation.

The above method, if reliable, would certainly lead to a more accurate determination of any scattering amplitude than if the far left-hand cut contributions were just ignored, but one would still have to treat the crossed channels as known. For instance, if the various low-energy, low angular-momentum partial-wave amplitudes for $\pi - N$ scattering were calculated and one thus found a bound state (nucleon) and a resonance (N^*), it would be only after one inserted them beforehand in the crossed $\pi - N$ channel. At this stage, however, if the criterion of self-consistency[5,6] is invoked,

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[1] S. C. Frautschi and J. D. Walecka, Phys. Rev. **120**, 1486 (1960). We follow the notation of this paper; note that our units are $\hbar = c = m_\pi = 1$.

[2] J. S. Ball and D. Wong, Phys. Rev. **133**, B179 (1964).

[3] L. A. P. Balazs, Phys. Rev. **128**, 1939 (1962).

[4] One will have to match the amplitude and the first $n-1$ derivatives, if the number of Balázs residues to be determined is n.

[5] G. F. Chew, *Proceedings of the International Conference on High-energy Nuclear Physics, Geneva, 1962*, edited by J. Prentki (CERN, Geneva, 1962), p. 525.

[6] G. F. Chew, Phys. Rev. Letters **9**, 233 (1962).